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Galton Board: Normal Distribution

intermediate

Stimulate problem solving	**silver**	**Stimulate entrepreneurship**	**silver**
Stimulate creativity	**bronze**	**Informal learning enviro.**	**gold**
Stimulate critical thinking	**silver**	**Technology use**	**bronze**
Stimulate group work	**platinum**		

Practicalities

Preparation: 1h

Duration:4- 4:45h

Group size range: 20 - 30
Ideal sub-group size: 3-5

Workshop made for: 14-16
Easily transferable to workshop for ages between: 14-18

Material needs:

- paper and pen
- weighing and measuring tools
- tools
- marbles (stell recommended)
- wooden board, screws, slats, bottle caps or other barriers
- boxes or other containers
- at least 1 data projector and PC with internet connection or PC for each group to experiment virtually with
- See “Preparation” for a full list of tools and materials needed

Environment FabLab necessary: NO

Educational area:

* Engineering
* Science
* Math
* Other

Prerequisites

Knowledge about relationship between cause and effect.
Knowledge about gravity.

(see box ‘content links’ below)

Preparation

You can use a wide variety of materials to create a Galton board, depending on its availability and processing capabilities. Our solution is one of the possible and practically every part is interchangeable. The range (number of rows) of the Galton board is also variable - the larger the more clear distribution should be.

In order to create Galton board precisely according to the plan you will need (for each group)

- backing board – as smooth as possible – size 8 0xm X 110 cm other dimensions are possible – need to adjust the size of the triangle
- slats for the edges of the board
- PET bottle caps 66* pieces**
- boxes to catch marbles 12* pieces
- box to create a funnel
- a board for the back support

Groups can share

- screws, screwdriver, glue, petty material
- marbles – steel recommended, as many as possible
- strong magnet – when steel marbles are used, useful when extracting the marbles

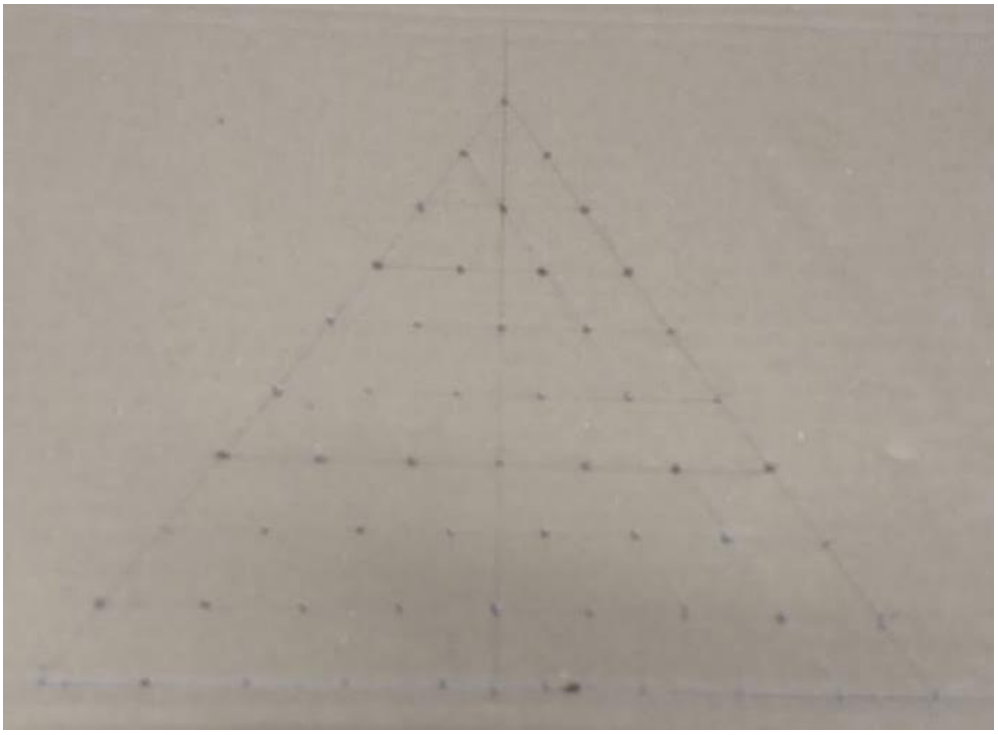
Older students can work with the drill without any problems – teacher will either create a template for them or consistently insist on pre-painting the necessary holes on the backing plate - any faulty holes will negatively affect the outputs.

* The numbers are based on our experience, there is no problem to adjust as needed

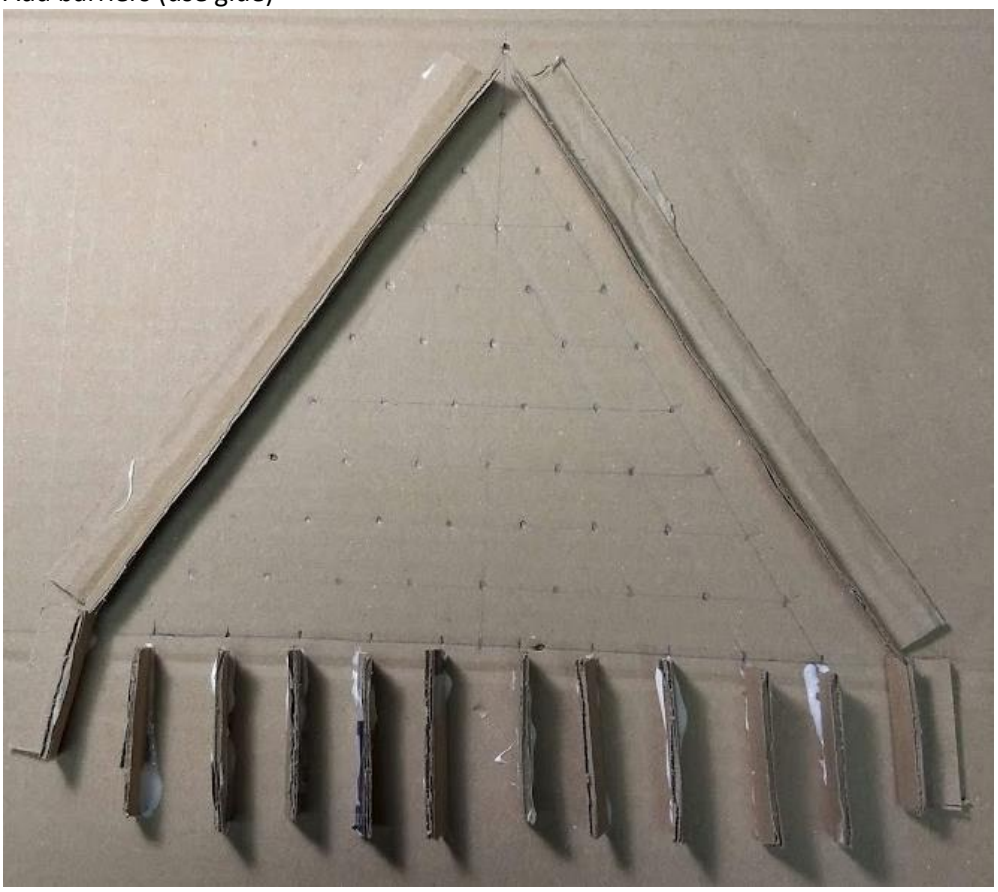
** PET bottle caps are not necessary components, they can be easily replaced by screws, pins, etc. - they were chosen for easy accessibility and to provoke an environmental view

Note: As a minimalist variant, we propose a flat carton into which the pins with larger heads are inserted - this board will be disposable, but it will also serve in case of lack of resources or time.

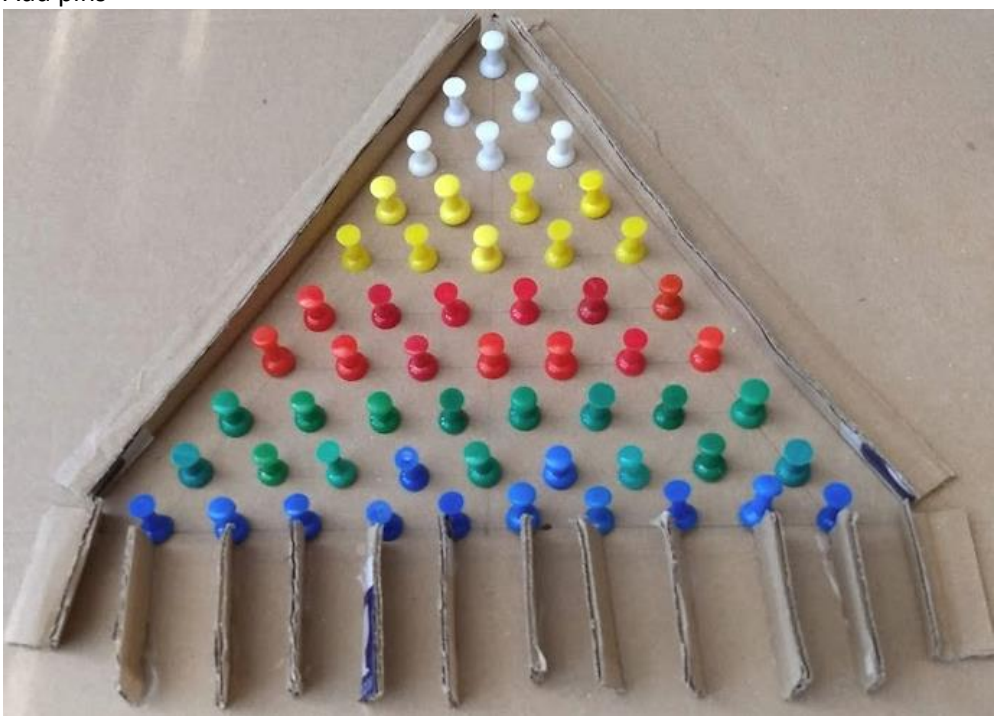
	The quick guide for a minimalist variant
You need	pasteboard, pencil, ruler, glue, pins, balls

Preparation	Draw position of the pins - you will need to find the best position of pins at pasteboard, try several options with at least four rows - draw pins position for the best results 

Add barriers (use glue)



Add pins



	And try your result  Voilá … simple galton desk is done!

Environment

If available we suggest to arrange the space with islands of tables spread around the room, so that both educators and participants can freely walk around them. Data projector and screen visible from every work place.

The theoretical part can be done in any classroom equipped with a data projector and for the practical part students can be moved to school workshops – based on school availability.

Workshop Guidelines

Phase 1: Orientation and Instruction Phase

Material needs:

Essential: PC with internet connection, worksheets, pen

***Optional*:**

Goals:

Skill Goals (**Blue**):

Content Goals (**Green**)
(C1) Analysing everyday objects and finding out how they work
(C2) Understanding the possibilities of movement in a predefined environment

Goals	Activities	Duration
	Motivationally, we show students (see links) an example of Galton. Using the online tool (see link 3) we will show them path of the marbles - randomness for small numbers - as the number increases, the rule begins to form. Using the online tool, we can also show them how the board will be affected by various settings.	10’
C1, C2	Then we let the students prepare a Worksheet 2. It would be appropriate for them to realize / deduce that multiple paths are moving towards mean values - and why.	30’
	We will familiarize them with the aim of the workshop - to create our own Galton board and to verify the assumptions obtained by using a worksheet and an online tool in practice. At the same time, it acquaints pupils with the material available to me.	5’
	Optional (if you are using bottle caps) it is worth including an ecological moment that will correct the “mode” of collecting bottle caps (at least in the Czech Republic): - one cap weighs an average of 2 grams - the average purchase is EUR 0,22 per kilogram	10’

	We need at least 2 314 caps to get 1 EUR ”help”. At the price of bottled water approx. 0,18 EUR (in Czechia) / bottle we need to spend more than 416 EUR to get 1EUR donate (instead of donating the 1 EUR directly) and this does not include the cost of producing, storing, transporting and processing waste (bottles) - for various cola drinks, the 1 EUR of “help” will cost us at least EUR 1 560. Once again, instead of spending 1 560 EUR, it is better directly donate 1 EUR and avoid buying beverages high in sugar and other unnecessary substances Collection of caps is suitable for school experiments but it is never a charity.	

Phase 2: Design Phase

Material needs:

Essential: sketching materials: paper, pens, pencils, ...

***Optional*:** ruler, measuring tape

Goals:

Skill Goals (**Blue**):
(S1) Abstraction of an idea to a 2-dimensional sketch
(S2) Working in a group
(S3) Communication
(S4) Providing feedback to someone else´s idea

Content Goals (**Green**)
(C1) Bounce effect of material

Goals	Activities	Duration
S1, S2 C1	**Make a plan** Each group needs to brainstorm and draw a sketch of their Galton board Each group must decide: - What we need? - How do we split our work? - What problems can we face? And how do we prevent them? In addition to the below, it is necessary to somehow define the space on the sides – the laths will do just fine. Another problem is the tilt of the board - with a higher tilt the movement will be faster - and the marbles will get trapped in the crucible at the bottom BUT we are at risk of "out-popping"of marbles, not only the marbles do not run around the caps from the left / right, but they can jump over them and completely destroy the experiment. Definitely there will be a problem how to solve the spacing between individual caps - too small / too large will adversely affect the result There are several possibilities how to work - Letting the students work by themselves - in this case it is possible to skip the next section, but it is necessary to take into account that even the functionality of the Galton board will be only due to chance. - We consider it ideal to have students estimate the correct values by experiment Tip: We have proved that an isosceles triangle with a base of 7cm and a height of 5cm to be the best - but these values will change with the material used.	30'

	We recommend that you guide your students to create a template to speed up their work.	
S4, S5	***Optional*** Present every group with a problem – how to solve the gap between the caps? The best way is to try to make a small cut of the Galton board - for example, only three rows, do not screw the caps, but only stick / hold at different distances and try to find out if the "passage is random". To do this, one marble is enough for the group - if, after several attempts, they are able to correctly estimate where the marble will be released at the next attempt - the spacing is wrong and another option must be tried - The marble goes to the edges - the caps are too close to each other - The marble moves through the center - the caps are too far apart	20’

Phase 3: Making Phase

Material needs:

Essential: board, laths, caps, screws, cups, pencil, auger

***Optional*:** drill, ruler, plexi glass, marbles

Goals:

Skill Goals (**Blue**):
(S1) Trial and error/ Deal with failure
(S2) Working in group
(S3) Asking for help
(S4) Communication

Content Goals (**Green**)
(C1) Work with material

Goals	Activities	Duration
S1, S2 S3, C1	**Let's build!** Each group receives a board and drawing equipment - make sure that they first prepare with a pencil the placement of laths and caps on the board (working with a ruler, determining the right angle). If the students are given the board already pre-drawn / pre-drilled, this phase is not necessary. The trainer should encourage the students to create a template and not measure each item individually. 	1h30'

Then pupils drill holes for screwing the caps and laths and screw them in. It is advisable to lead students to a division of labour - someone prepares holes in the board, other holes in the caps, others screws.

At this moment we have a simple Galton board, which will work with great limitations - the slope must be small, we drop / catch (count) marbles manually. All this is unsuitable for tracking probability = working with large values.

	We save the manual work (holding) by the stand in the back of the Galton board. One board is really enough, the base is wide enough to ensure stability. 	

For meaningful work it is necessary to place one trap = cup under each exit.

Not a necessary but appropriate component is the marbles tray at the top - we do not have to release the balls manually, but we can run them at the same time. This is not necessary for the final evaluation, but it speeds up the work and is also suitable for triggering the

“WOW effect” in Phase 4

It is also advisable to cover a large area (especially in the upper part) with plexiglass or something similar - to prevent the marbles from popping when dropped at high tilt. With a small tilt, this is not necessary - it is necessary to keep this in mind when catching the marbles - when the marble may not have enough energy to "skip" the rim of the cup. Anyway, it is up to the teacher / students to think about it in Phase 2.

	Optional Encourage students in their creativity and allow them some time to spoil their creations - hopper, containers, edges are a place where students can use their creativity to relax after and before the	15 ,

more mentally demanding activities.

Phase 4: Operational Phase

Material needs:

Essential: Galton board, marbles, paper, pencil

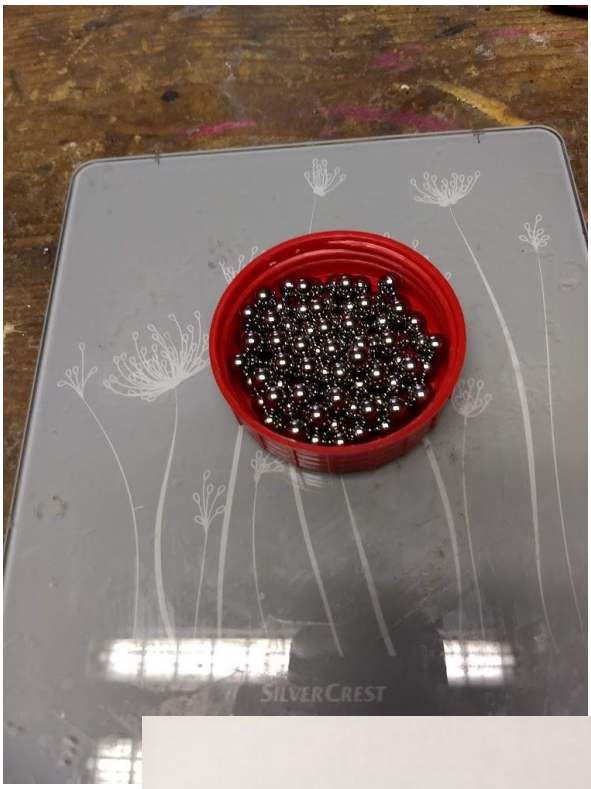
***Optional*:** ruler, scale, magnet

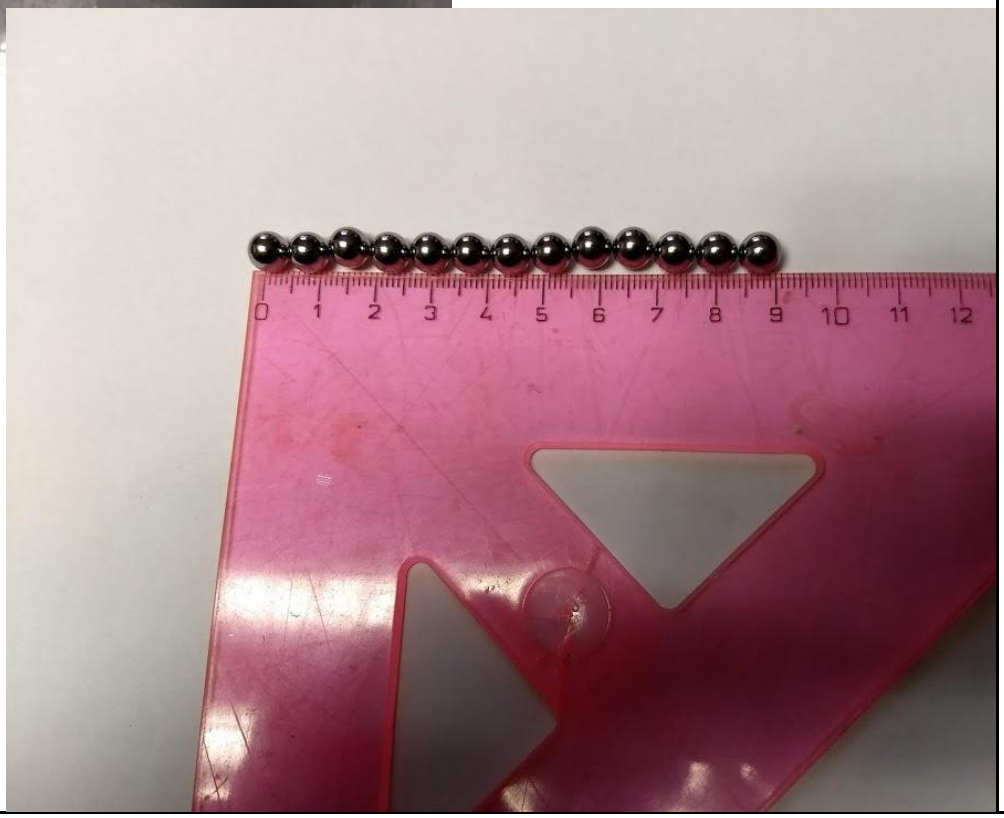
Goals:

Skill Goals (**Blue**):
(S1) Trial and error/ Deal with failure
(S2) Working in group

Content Goals (**Green**)
(C1) Research based learning

Goals	Activities	Duration
S1, S2	**Test and troubleshooting** Try running a few marbles on the created board and answer the questions (which indicate the problem). - Aren't they always going the same way? Why? - Can you guess where the marble will go? Why? - Are the marbles out of the way? In case of a problem, try to remove it - by changing the inclination of the board, adjusting the spacing between the individual caps. Tips for educators: - Is the board tilted more to one side? - Is the tilt of the board unnecessarily large? - Is the spacing between the caps too large / small?	10’
C1	**Let's try** At this stage, students try to release a large number of marbles down the Galton board and note down the number of marbles in each cup after passing through the entire board. With large numbers of marbles, the counting (although possible) is ineffective. Therefore, it is good to use the knowledge of Phase 2 and "count" marbles by weighing or measuring. It is advisable to repeat the experiment several times so that the results obtained can help correct generalization in Phase 5. Noting down values is very important for a successful Phase 5. It is virtually certain that the results will not be “ideal”, students should be reminded of that, the existence of many variables and the fact that the probability works on large numbers. The online tool can be used again by success (see link 3)	30’


SILVERCREST


0 1 2 3 4 5 6 7 8 9 10 11 12

Phase 5: Evaluation Phase

Material needs:

Essential: pen and paper

***Optional*:**

Goals:

Skill Goals (**Blue**):
(S1) Reflecting on the project
(S2) Communication

Content Goals (**Green**)

Goals	Activities	Duration
S1	**Reflections** Gather all groups together and ask the students to present the individual group results. Ask each participant to reflect on their own about the whole process. Provide them with some guiding questions like: - What worked well in the reaction and in your station? Why? - What was the most difficult thing to achieve as a group? Why? - How does the actual number of marbles in each cup differ from the expected computational model? Why? - What would the result look like if we made the board bigger (with a larger number of rows)? - Can you estimate the result in individual cups for different number of marbles? - What improvements would you suggest?	20’

S2	**Sharing** Let the participants share their reflection in group. Can they generalize the findings?	15’

Pedagogical tips

We definitely recommend to prepare a sample of the online version of the Galton board (see the links) for discussion on work with large numbers.
It is clear (and should be mentioned) that no board will come out of the ideal distribution (it is advisable to involve students in the discussion and searching for reasons), for example, this may be due to:

- Board irregularities
- Uneven caps
- Inaccurate spacing of caps
- Small number of attempts
- Low / high lowering speeds) tilt angle)

We invite you to try the activity yourself before doing it so you can anticipate where kids may get stuck or need guidance. The mechanism as such is not difficult to build, but will be greatly influenced by any inaccuracies in the measurement.

How to transfer to non-Fablab environment

Transfer to nonfablab environment is very feasible, as long there is enough space, tables and tools available.

The materials used for construction of the Galton board is dependent on the available resources.

Evaluation of achievements

At the end of the workshop you can give the different groups achievements. For example, for:

- The most beautiful Galton board
- The most accurate Galton board
- The team with the most accurate description of outcomes
- The best team
- The team that bests simulated Gaus's distribution

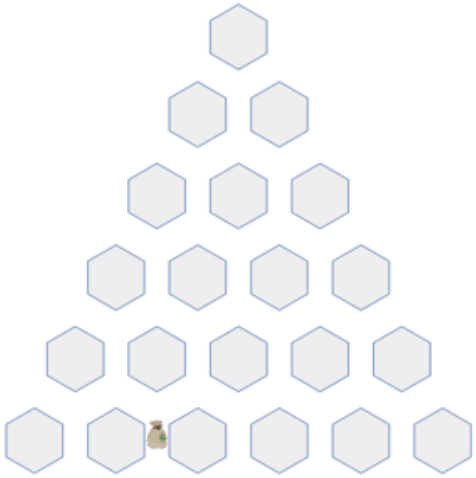
Content links

Some interesting links that can inspire you

- Galton board
- Bean machine
- Galton board online

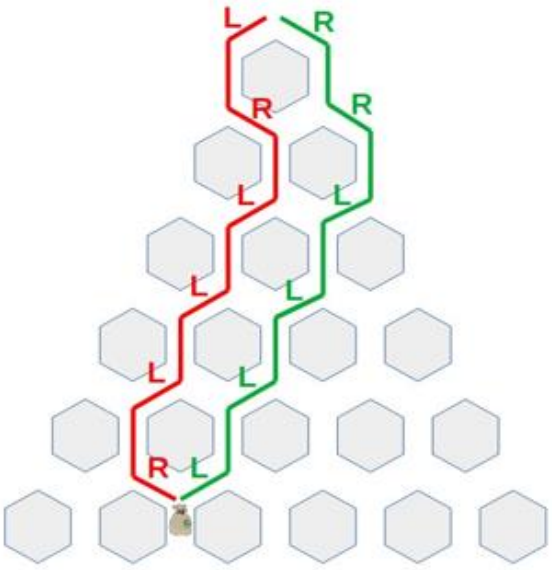
WORKSHEET

Pirates want to hide their treasure on a secret island. In the underground there is a cave with many crossroads, where one can get easily lost. At each crossroad you can go either right or left. The map for the treasure is shown in the next picture.



There are several ways how to reach the treasure. It depends on whether we turn left or right at the crossroads.

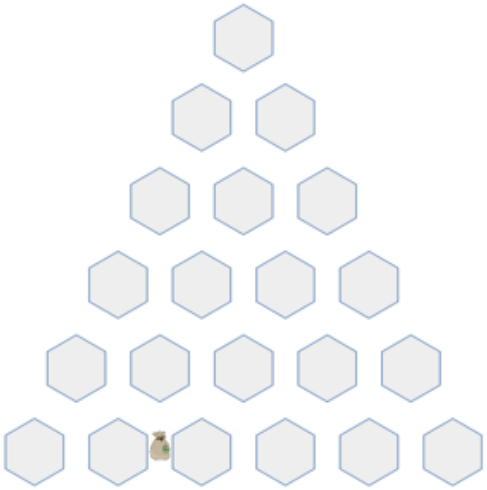
There are two ways that go through the maze to the treasure.



At each crossroad we decide between turning left or right and type L or R to the table accordingly.

1.crossroad	2.crossroad	3.crossroad	4.crossroad	5.crossroad	6.crossroad
L	R	L	L	L	R
R	R	L	L	L	L

Find another way to the treasure and add it to the table.

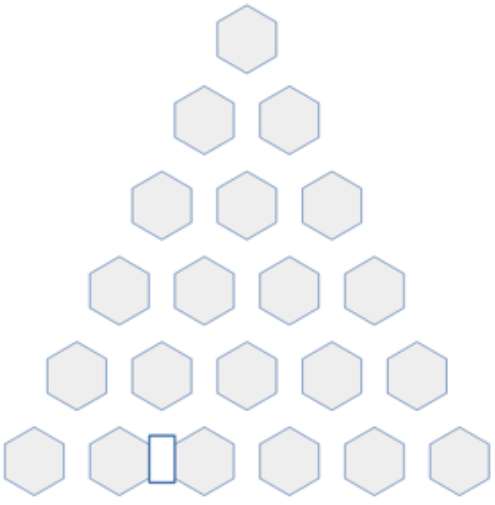


1.crossroad	2.crossroad	3.crossroad	4.crossroad	5.crossroad	6.crossroad
L	R	L	L	L	R
R	R	L	L	L	L

Write down all the ways which lead to the treasure.

1.crossroad	2.crossroad	3.crossroad	4.crossroad	5.crossroad	6.crossroad
L	R	L	L	L	R
R	R	L	L	L	L

Enter the total number of possible ways in the box next to the treasure.



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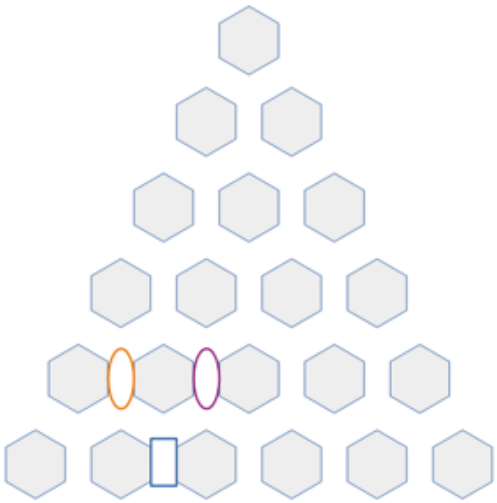
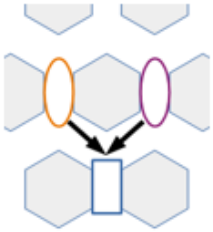


Let's think further about what would happen if pirates hid the treasure elsewhere. In the tables below write down all the ways you can take to get to the orange elipse and then to the purple ellipse. Then write the number of ways into ellipses.

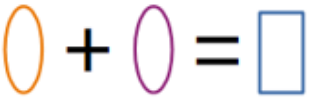
1.crossroad	2.crossroad	3.crossroad	4.crossroad	5.crossroad

1.crossroad	2.crossroad	3.crossroad	4.crossroad	5.crossroad

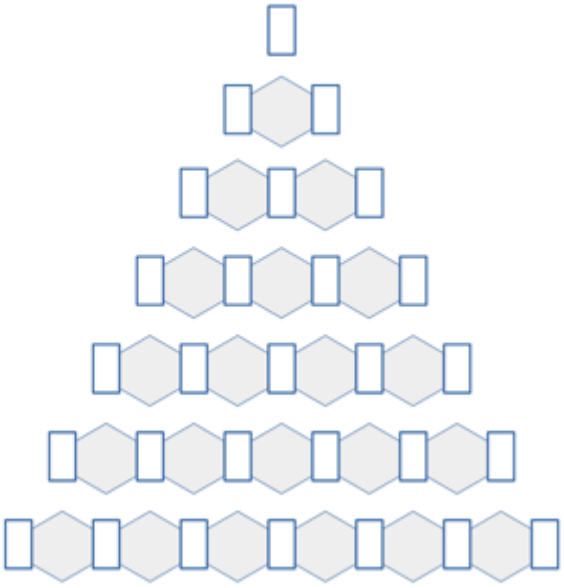
There are only two ways to reach the blue rectangle:
From the orange ellipse or from the purple ellipse.



The number of ways to the blue rectangle can therefore be calculated as



Fill in the number of ways to all points of the maze.



Such a scheme is called Pascal´s triangle.

Let's imagine that our maze will be a little bigger. How many times is there a chance that we will find an orange rectangle rather than a purple rectangle in case of a random treasure hunt?